Int. J. Heat Mass Transfer. Vol. 19, pp. 1117-1120. Pergamon Press 1976. Printed in Great Britain

INJECTION EFFECT IN MULTICOMPONENT BOUNDARY LAYER

S. ENDRÉNYI and B. PALÁNCZ
Refrigerating Works, Budapest, Hungary

and

Institute for Industrial Economy Organization and Computation Technique, Budapest, Hungary

(*Received* 22 *March* 1976)

Abstract—A model for determining transport coefficients in the case of simultaneous heat, mass and momentum transfer in multicomponent boundary-layer flow along a flat interface has been delivered. The equations of the model were solved by the quasilinearisation technique. The practical utility and rapidity of the method is well illustrated. The computational studies of dimensionless transfer coefficients showed that the effects of multicomponent diffusion as well as injection caused by mass flux at the interface are not neglectable.

NOMENCLATURE

a, coefficients defined in (5);
c, specific heat;
c, concentration;
D, diffusivity;
f, function;
i, enthalpy;
h, heat-transfer coefficient;
k, thermal conductivity;
N, mass flux;
P, total pressure;
q, heat flux;
r, heat of evaporation;
R, universal gas constant;
T, temperature;
u, velocity;
v, velocity;
x, variables defined by (15);
x, coordinate;
y, coordinate;
ϕ, diffusivity;
ρ, density;
μ, dynamic viscosity;
ν, kinematic viscosity;
ξ, friction coefficient;
τ, shear stress.

Dimensionless variables

Nu, Nusselt number;
Pr, Prandtl number;
Re, Reynolds number;
Sc, Schmidt number;
Sh, Sherwood number;
η, dimensionless coordinate;
ψ, stream function;
θ, dimensionless variables for intensive.

Subscripts

H, heat transfer;
i, ith component;
j, jth component;
ij, mutual value;
M, mass transfer;
0, interface or at zero temperature;
p, constant pressure;
∞, bulk.

INTRODUCTION

THERE are a lot of technological processes where heat, mass and momentum transfer occur simultaneously between different phases. In connection with these processes, it may be desirable to know the values of transfer coefficients. As is well known, these coefficients depend on each other. The problem becomes complicated when the phases contain several different components [1, 2].

This article deals with the determination of transport coefficients in the case of multicomponent boundary-layer flow above a flat interface.

The equations of the model can be solved by the quasilinearisation method. In this way, the dimensionless form of the transfer coefficients can be expressed as in the case of two components.

As a result, the computational studies of this model show the effects of multicomponent diffusion as well as the injection caused by mass transfer at the interface. According to the numerical results, it is clear that these effects are not neglectable.

The convergence of the numerical procedure proved to be rapid in the practical view.

MATHEMATICAL MODEL

The model can be derived by the use of balance equations of momentum, heat and mass transport for two dimensional laminar boundary flow in the case of steady state.

The relations involve the following preconditions:

(1) velocity of flow has a potential function, (2) density of fluid is constant, (3) intensive properties of fluid (temperature and concentration) are constant in the bulk as well as at the interface, (4) other properties of fluid (viscosity, heat conductivity and diffusivity) are

considered at the average values of intensive properties as constant, (5) dissipation is neglectable and (6) chemical reaction does not occur.

The equation of conservation of mass:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0. \qquad (1)$$

The equation of conservation of momentum:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}. \qquad (2)$$

The equation of conservation for the jth component:

$$u\frac{\partial c_j}{\partial x}+v\frac{\partial c_j}{\partial y}=-\frac{\partial N_j}{\partial y}, \quad j=1,2,\ldots,n. \qquad (3)$$

The energy equation:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=a_H\left(\frac{\partial^2 T}{\partial y^2}-\sum_{j=1}^{n}a_{M_j}\frac{\partial N_j}{\partial y}\right), \quad j=1,2,\ldots,n \qquad (4)$$

where

$$a_H=\frac{k}{\rho c_p}, \quad a_{M_j}=\frac{i_j}{k}. \qquad (5)$$

The flux of the jth component can be expressed [3]:

$$N_j=-\sum_{i=1}^{n}\phi_{ji}\frac{\partial c_i}{\partial y}, \quad j=1,2,\ldots,n. \qquad (6)$$

Respecting assumptions mentioned before, the boundary conditions are:

$$\begin{aligned} &y=0, \quad u=0, \quad c_j=c_{j0}, \quad T=T_0\\ &v=v_0(x)\\ &y=\infty, \quad u=U_\infty, \quad c_j=c_{j\infty}, \quad T=T_\infty\\ &j=1,2,\ldots,n.\end{aligned} \qquad (7)$$

The component of velocity of flow in the y direction can be determined by the flux at the interface:

$$v_0(x)\sum_{j=1}^{n}c_{j0}=\sum_{j=1}^{n}N_j(x,0)=\sum_{j=1}^{n}N_{j0}(x)$$

and using equation (6):

$$v_0(x)=-\frac{\sum_{j=1}^{n}\sum_{i=1}^{n}\phi_{ji}\left(\frac{\partial c_i}{\partial y}\right)_{y=0}}{\sum_{j=1}^{n}c_{j0}}. \qquad (8)$$

The system of partial differential equations can be transformed into a system of ordinary differential equations. Let us consider the following dimensionless variable

$$\eta=\frac{y}{2}\sqrt{\left(\frac{U_\infty}{\nu x}\right)}$$

and the stream function

$$u=\frac{\partial\psi}{\partial y}, \quad v=-\frac{\partial\psi}{\partial x}$$

where

$$\psi=\sqrt{(U_\infty \nu x)}\cdot f(\eta)$$

which satisfies the continuity equation.

Introducing dimensionless variables for intensive properties:

$$\theta_H=\frac{T-T_0}{T_\infty-T_0}$$

$$\theta_{M_j}=\frac{c_j-c_{j0}}{c_{j\infty}-c_{j0}}, \quad j=1,2,\ldots,n$$

so that dimensionless forms of equations (2)–(6) are:

$$f'''+ff''=0 \qquad (9)$$

$$\sum_{i=1}^{n}s_{ji}\theta''_{M_i}+p_jf\theta'_{M_j}=0, \quad j=1,2,\ldots,n \qquad (10)$$

and

$$\theta''_H+\sum_{j=1}^{n}z_j\theta''_{M_j}+Prf\theta'_H=0. \qquad (11)$$

Boundary conditions:

$$\begin{aligned} &\eta=0, \quad f=f_0, \quad \theta_{M_j}=0, \quad \theta_H=0\\ &f'=0\\ &\eta=\infty, \quad f'=2, \quad \theta_{M_j}=1, \quad \theta_H=1, \quad j=1,2,\ldots,n\end{aligned} \qquad (12)$$

and according to equation (8):

$$f_0=\sum_{j=1}^{n}w_j\theta'_{M_j}(0). \qquad (13)$$

Choosing the kth component as a key-component, the coefficients of equations (10), (11) and (13) are:

$$\begin{aligned} s_{ji}&=\frac{c_{i\infty}-c_{i0}}{c_{k\infty}-c_{k0}}\frac{\phi_{ji}}{\nu}, \quad j=1,2,\ldots,n\\ z_j&=\frac{c_{j\infty}-c_{j0}}{T_\infty-T_0}\sum_{i=1}^{n}a_{M_i}\phi_{ij}, \quad j=1,2,\ldots,n\\ w_j&=\frac{c_{j\infty}-c_{j0}}{\sum_{i=1}^{n}c_{i0}}\frac{\sum_{i=1}^{n}\phi_{ij}}{\nu}, \quad j=1,2,\ldots,n\\ p_j&=\frac{c_{j\infty}-c_{j0}}{c_{k\infty}-c_{k0}}, \quad j=1,2,\ldots,n.\end{aligned} \qquad (14)$$

SOLUTION OF THE EQUATIONS

This boundary value problem can be solved numerically by the quasilinearization method [4]. For the sake of this solution, we transformed the system of equations into the first order system. The new variables are:

$$\begin{aligned} x_1&=f\\ x_2&=f'\\ x_3&=f''\\ x_{2j+2}&=\theta_{M_j}, \quad j=1,2,\ldots,n\\ x_{2j+3}&=\theta'_{M_j}, \quad j=1,2,\ldots,n\\ x_{2n+4}&=\theta_H\\ x_{2n+5}&=\theta'_H.\end{aligned} \qquad (15)$$

So the new system is:

$$\begin{aligned} x'_1&=x_2\\ x'_2&=x_3\\ x'_3&=-x_1x_3\\ x'_{2j+2}&=x_{2j+3}, \quad j=1,2,\ldots,n\end{aligned}$$

$$x'_{2j+3} = -x_1 \sum_{i=1}^{n} s_{ji}^{-1} x_{2i+3} p_i, \quad j = 1,2,\ldots,n \quad (16)$$

$$x'_{2n+4} = x_{2n+5}$$

$$x'_{2n+5} = x_1\left(\sum_{j=1}^{n} z_j \sum_{i=1}^{n} s_{ji}^{-1} x_{2i+3} - Pr x_{2n+5}\right)$$

where

$$\{s_{ji}^{-1}\}_{i,j=1}^{n} \{s_{ji}\}_{i,j=1}^{n} = \{\delta_{ji}\}_{i,j=1}^{n}.$$

Boundary conditions:

$$\begin{aligned} x_1(0) &= f_0 \\ x_{2j+2}(0) &= 0, \quad j = 1,2,\ldots,n \\ x_{2n+4}(0) &= 0 \\ x_2(\infty) &= 2 \\ x_{2j+2}(\infty) &= 1, \quad j = 1,2,\ldots,n \\ x_{2n+4}(\infty) &= 1 \end{aligned} \quad (17)$$

where

$$f_0 = \sum_{j=1}^{n} w_j x_{2j+3}(0).$$

Now this split-boundary value problem can be solved by the quasilinearization method directly.

DETERMINATION OF TRANSPORT COEFFICIENTS

Using the solution of the system described above, we can express the relations among the dimensionless forms of transport coefficients for the multicomponent system.

The friction coefficient characterizing momentum transport is:

$$\xi(x) = \frac{\tau_0}{\rho U_\infty^2}$$

and

$$\tau_0 = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0}$$

so

$$\xi(x) = \tfrac{1}{4} f''(0)\sqrt{\left(\frac{v}{xU_\infty}\right)}$$

or (18)

$$\xi(x) = \frac{x_3(0)}{4} Re^{-1/2}(x).$$

Due to the term of mass-transfer coefficient

$$k_j = \frac{N_j}{c_{j0} - c_{j\infty}}, \quad j = 1,2,\ldots,n.$$

The dimensionless relations describing mass transfer:

$$\frac{Sh_j(x)}{\sqrt{[Re(x)]}} = \tfrac{1}{2}\left(\sum_{i=1}^{n} s_{ji} Sc_{ji} x_{2i+3}(0)\right), \quad j = 1,2,\ldots,n \quad (19)$$

where

$$Sc_{ji} = \frac{v}{D_{ji}}.$$

Relations for heat transfer:

$$\frac{Nu(x)}{\sqrt{[Re(x)]}} = \tfrac{1}{2}\left(x_{2n+5}(0) + \sum_{i=1}^{n} z_i x_{2i+3}(0)\right) \quad (20)$$

and the term of heat-transfer coefficient

$$h(x) = \frac{q(x)}{T_0 - T_\infty}$$

where

$$q(x) = -k\left(\frac{\partial T}{\partial y}\right)_{y=0} + \sum_{i=1}^{n} N_j i_j. \quad (21)$$

As illustration of the model, these relations were applied to a gas flow consisting of three components, one of which is inert.

Let us consider the component with index 1 as the key component. So equations for the case of three components are:

$$\begin{aligned} x'_1(\eta) &= x_2 \\ x'_2(\eta) &= x_3 \\ x'_3(\eta) &= -x_1 x_3 \\ x'_4(\eta) &= x_5 \\ x'_6(\eta) &= x_7 \\ x'_5(\eta) &= -x_1(s_{11}^{-1} x_5 p_1 + s_{12}^{-1} x_7 p_2) \\ x'_5(\eta) &= -x_1(s_{11}^{-1} x_5 p_1 + s_{12}^{-1} x_7 p_2) \\ x'_{10}(\eta) &= x_{11} \\ x'_{11}(\eta) &= x_1[z_1(s_{11}^{-1} x_5 + s_{12}^{-1} x_7) \\ &\quad + z_2(s_{21}^{-1} x_5 + s_{22}^{-1} x_7) - Pr x_{11}] \end{aligned} \quad (22)$$

where s_{ji}^{-1} are the elements of the inverse of the following matrix:

$$\{s_{ji}\}_{i,j=1}^{n} = \frac{1}{v}\begin{bmatrix} \phi_{11} & \phi_{12}\dfrac{c_{2\infty} - c_{10}}{c_{1\infty} - c_{10}} \\ \phi_{21} & \phi_{22}\dfrac{c_{2\infty} - c_{10}}{c_{1\infty} - c_{10}} \end{bmatrix} \quad (23)$$

and

$$\begin{aligned} z_1 &= \frac{c_{1\infty} - c_{10}}{T_\infty - T_0}(a_{M1}\phi_{11} + a_{M2}\phi_{21}) \\ z_2 &= \frac{c_{2\infty} - c_{20}}{T_\infty - T_0}(a_{M1}\phi_{12} + a_{M2}\phi_{22}) \\ p_1 &= 1 \\ p_2 &= \frac{c_{2\infty} - c_{20}}{c_{1\infty} - c_{10}}. \end{aligned} \quad (24)$$

Boundary conditions:

$$\eta = 0, \quad x_1 = f_0, \quad x_4 = 0, \quad x_6 = 0, \quad x_{10} = 0$$
$$x_2 = 0$$
$$\eta = \infty, \quad x_2 = 2, \quad x_4 = 1, \quad x_6 = 1, \quad x_{10} = 1$$

where

$$f_0 = w_1 x_5(0) + w_2 x_7(0)$$

and

$$w_1 = \frac{c_{1\infty} - c_{10}}{c_{10} + c_{20}} \frac{\phi_{11} + \phi_{21}}{v}$$

$$w_2 = \frac{c_{2\infty} - c_{20}}{c_{10} + c_{20}} \frac{\phi_{12} + \phi_{22}}{v}.$$

The dimensionless equations are:

$$\xi(x) = \tfrac{1}{4} x_3(0) Re^{-1/2}(x)$$

$$\frac{Sh_1(x)}{\sqrt{[Re(x)]}} = \tfrac{1}{2}[s_{11} Sc_{11} x_5(0) + s_{12} Sc_{12} x_7(0)]$$

$$\frac{Sh_2(x)}{\sqrt{[Re(x)]}} = \tfrac{1}{2}[s_{21} Sc_{21} x_5(0) + s_{22} Sc_{22} x_7(0)]$$

$$\frac{Nu(x)}{\sqrt{[Re(x)]}} = \tfrac{1}{2}[x_{11}(0) + z_1 x_5(0) + z_2 x_7(0)].$$

The system of equations (22) were solved with the following data:

$$P = 1.0\,\text{atm}$$
$$R = 0.082\,\text{atm/K kmól}$$
$$D_{AC} = 0.0476\,\text{m/h}$$
$$D_{AB} = 0.0575\,\text{m/h}$$
$$D_{BC} = 0.106\,\text{m/h}$$
$$Pr = 0.704$$
$$C_{pC} = 0.248\,\text{kcal/kg}^\circ\text{C}$$
$$r_{A0} = 220\,\text{kcal/kg}$$
$$r_{B0} = 597\,\text{kcal/kg}$$
$$c_{pA} = 0.36\,\text{kcal/kg}^\circ\text{C}$$
$$c_{PB} = 0.46\,\text{kcal/kg}^\circ\text{C}$$
$$C_A = 0.003\,\text{kmol/m}^3$$
$$C_B = 0.0048\,\text{kmol/m}^3$$
$$T = 372\,\text{K}.$$

These results are compared with the results given by equations for a two-components system where one component is inert [5].

$$\begin{aligned} \xi(x) &= 0.332 Re^{1/2}(x) \\ Sh_1(x) &= 0.332 Re^{1/2}(x) Sc_{13}^{1/3} \\ Sh_2(x) &= 0.332 Re^{1/2}(x) Sc_{23}^{1/3} \\ Nu(x) &= 0.332 Re^{1/2}(x) Pr^{1/3}. \end{aligned} \tag{25}$$

These equations do not take into consideration the effect of injection caused by mass transfer.

For separating effects of injection and multicomponent transport, we also solved equations (22) under condition $v_0(x) = 0$.

Comparisons of the results are illustrated in Table 1.

Table 1

	From equation (22) $v_0(x) \neq 0$	From equation (22) $v_0(x) = 0$	From equation (25)
$\frac{\xi(x)}{\sqrt{[Re(x)]}}$	0.223	0.332	0.332
$\frac{Sh_1(x)}{\sqrt{[Re(x)]}}$	0.328	0.502	0.402
$\frac{Sh_2(x)}{\sqrt{[Re(x)]}}$	0.303	0.648	0.305
$\frac{Nu(x)}{\sqrt{[Re(x)]}}$	0.095	0.148	0.296

REFERENCES

1. B. Gai-Or and D. Weihs, Thermodynamic coupling in boundary layers by a rapid coupling-factor estimate, *Appl. Scient. Res.* **29**, 44 (1974).
2. Y. Taitel and A. Tamir, Multicomponent boundary layer characteristics—use of the reference state, *Int. J. Heat Mass Transfer* **18**, 123–129 (1976).
3. E. R. G. Eckert and R. M. Drake, *Heat and Mass Transfer Analysis*. McGraw-Hill, New York (1972).
4. G. Beveridge and R. Schechter, *Optimization: Theory and Practice*. McGraw-Hill, New York (1970).
5. W. M. Rohsenow and J. P. Hartnett, *Handbook of Heat Transfer*. McGraw-Hill, New York (1972).

EFFET DU SOUFFLAGE DANS LA COUCHE LIMITE A PLUSIEURS CONSTITUANTS

Résumé—On présente un modèle permettant de déterminer les coefficients de transport dans une couche limite à plusieurs constituants le long d'un interface plan avec transfert simultané de chaleur, de masse et de quantité de mouvement. Les équations du modèle ont été résolues par une méthode de quasi-linéarisation. L'utilité pratique de la méthode et sa rapidité sont mis en évidence. Les études numériques sur les coefficients de transfert adimensionnels ont montré que les effets de la diffusion des constituants ainsi que du soufflage produit par le flux massique à l'interface ne sont pas négligeables.

DER EINBLASEFFEKT BEI MEHRKOMPONENTEN-GRENZSCHICHTEN

Zusammenfassung—Es wurde ein Modell entwickelt zur Bestimmung der Transportkoeffizienten für den Fall gleichzeitigen Wärme-, Stoff-und Impulsaustausches in Mehrkomponenten-Grenzschicht strömungen entlang ebener Grenzflächen. Die Gleichungen des Modells wurden mit Hilfe der Methode der Quasilinearisation gelöst. Die praktische Eignung der Methode, die sehr schnell zum Ziel führt, wird demonstriert. Die rechnerische Untersuchung der dimensionslosen Transportkoeffizienten zeigte, daß der Einfluß der Mehrkomponentendiffusion sowie des Einblasens infolge eines Massenstromes an der Grenzfläche nicht vernachlässigbar ist.

ВЛИЯНИЕ ВДУВА В МНОГОКОМПОНЕНТНОМ ПОГРАНИЧНОМ СЛОЕ

Аннотация — Представлена модель для определения коэффициентов переноса в случае совместного переноса тепла, массы и количества движения в многокомпонентном пограничном слое на плоской границе раздела. Уравнения для модели решены методом квазилинеаризации. Показано, что данный метод позволяет быстро и легко выполнять расчеты. Расчеты безразмерных коэффициентов переноса показали, что влиянием многокомпонентной диффузии, а также влияниемвдува за счет массового расхода на поверхности раздела, нельзя пренебречь.